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2002 J. Phys.: Condens. Matter 14 3553

(http://iopscience.iop.org/0953-8984/14/13/313)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **14** (2002) 3553–3564 PII: S0953-8984(02)30378-3

Shock waves in crystalline dielectrics at low temperature

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Received 25 October 2001, in final form 7 February 2002
Published 22 March 2002
Online at stacks.iop.org/JPhysCM/14/3553

Abstract
The aim of this work is to study shock wave propagation in dielectric undeformable isotropic crystals at very low temperature ($T < 20$ K). Starting from a model based on extended irreversible thermodynamics and using the Lax and entropy growth selection rules, it is shown that not only the classical (hot) shocks but also so-called cold shocks are physically admissible. Cold shocks are characterized by the property that the body is cooled after the passage of the wavefront.

1. Introduction

It is well known that Fourier's law describes heat conduction accurately in most engineering applications. Unfortunately it predicts that thermal disturbances arising in a medium will travel with infinite velocity at high frequencies.

Moreover, some experimental results are not compatible with the Fourier law. In particular it was observed in 1944 [1] that heat could propagate as a true wave, called second sound, in superfluid helium II, at temperatures ranging from 1 to 2.2 K. The most popular modelling of second-sound propagation in He II was the two-fluid model introduced by Landau [2] and generalized by Kalatnikov [3].

The first theoretical formulation of heat conduction with finite wave speed was proposed by Cattaneo [4] who modified Fourier's law by introducing an inertial non-steady term. Combining Cattaneo's equation with the energy balance results in a hyperbolic evolution equation for the temperature field, allowing for a wave travelling with finite velocity.

Much effort has been devoted to finding evidence for temperature waves in solids. Heat pulse propagation techniques at low temperature were developed during the 1970s, and second

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sound was detected in dielectric crystals of high purity, like NaF, Bi, He-3 and He-4 in low-temperature ranges [5,6]. This is why the present model is particularly well suited to describing heat transport for this specific class of crystals.

The theoretical study of second-sound propagation in crystals at low temperature (<20 K) has received much attention during the last few decades and was studied by, among others, the present authors [7, 8] who based their analysis on extended irreversible thermodynamics (EIT) (see e.g. [9] for a review). The basic idea underlying EIT is to extend the space of the classical variables (like mass, momentum and energy) by including extra variables, essentially the fluxes of mass, momentum and energy, that vanish at equilibrium.

Within the framework of EIT, heat conduction in rigid bodies is described by the temperature T (classical variable) and the heat flux vector $\boldsymbol{q}$ (extra variable). In the present work, devoted to the problem of heat transport in rigid bodies, we generalize this choice by selecting $\boldsymbol{c} = \boldsymbol{q}/\lambda$ as a new variable, with $\lambda(T)$ the temperature-dependent heat conductivity. This choice is rather natural as it contains the non-equilibrium flux, here $\boldsymbol{q}$, and the corresponding transport coefficient, here λ. (Of course, this choice could be generalized to different situations: mass diffusion, electrical transport, hydrodynamics, …, wherein the extra variables will be selected as the ratios of the diffusion flux and diffusion coefficient, the electrical current and the electrical conductivity, ….) Moreover, this new variable is well suited to discussions of the problem of shock waves as it leads directly to an evolution equation in conservative form. In addition, with this variable $\boldsymbol{c}$, we are able to derive directly Grioli's equation [10–12].

After justifying the present model in the framework of non-equilibrium thermodynamics (section 2), we establish in section 3 some general results about wave and shock propagation in hyperbolic systems. In section 4, we establish the Rankine–Hugoniot admissibility conditions for the existence of shocks and we study the stability of the solutions by means of both the Lax criterion and the entropy growth rule. An important result of this work is that the model predicts not only the existence of classical (hot) shocks but also so-called cold shocks whose effect is to cool the body after the passage of the front. Conclusions and a comparison with other works are presented in section 5.

2. Non-equilibrium thermodynamic formulation of heat conduction

From the general statements of EIT, it is admitted that the space of state variables is formed by the union of the classical variables (here the internal energy u per unit volume or the temperature T) and the corresponding flux (here the heat flux $\boldsymbol{q}$). However, for generality, we will select the extra variable $\boldsymbol{c} = \boldsymbol{q}/\lambda$ wherein $\lambda(T)$ is the heat conductivity depending only on T.

The choice of $\boldsymbol{q}/\lambda$ rather than $\boldsymbol{q}$ is dictated by our aim of concentrating into one single variable all the non-equilibrium transport properties; another aim is to obtain evolution equations written in conservative form. To summarize, we select as state variables the set

$$u, \boldsymbol{c}\ (=\boldsymbol{q}/\lambda). \tag{1}$$

The evolution equation for u is the classical energy balance

$$\partial_t u + \nabla \cdot \boldsymbol{q} = 0. \tag{2}$$

The evolution equation for $\boldsymbol{c}$ will be determined from the general procedure followed in EIT. The starting point is to derive the expression for the entropy production defined through

$$\sigma^s = \partial_t s + \nabla \cdot \boldsymbol{J}^s \geqslant 0, \tag{3}$$

where σ^s is positive definite by virtue of the second law of thermodynamics, s is the entropy per unit volume and $\boldsymbol{J}^s$ is the entropy flux. To calculate σ^s, we need an expression for s in terms of the basic variables, i.e. $s = s(u, c^2)$ or in differential form:

$$\mathrm{d}s = \frac{\partial s}{\partial u}\,\mathrm{d}u + 2\frac{\partial s}{\partial c^2}\boldsymbol{c}\cdot\mathrm{d}\boldsymbol{c}. \tag{4}$$

As usual, we define the non-equilibrium temperature by

$$T^{-1} = \frac{\partial s}{\partial u}. \tag{5}$$

Substituting (5) in (4) yields the generalized Gibbs equation

$$\mathrm{d}s = T^{-1}\,\mathrm{d}u + 2\frac{\partial s}{\partial c^2}\boldsymbol{c}\cdot\mathrm{d}\boldsymbol{c}. \tag{6}$$

In the classical theory of irreversible processes only the first term on the right-hand side of (6) is present. Using the result (6), expression (3) for σ^s takes the form

$$T\sigma^s = \partial_t u + 2T\frac{\partial s}{\partial c^2}\boldsymbol{c}\cdot\partial_t\boldsymbol{c} + T\boldsymbol{\nabla}\cdot\boldsymbol{J}^s \geqslant 0, \tag{7}$$

or, by virtue of energy balance (2),

$$T\sigma^s = \boldsymbol{\nabla}\cdot(T\boldsymbol{J}^s - \lambda\boldsymbol{c}) + 2T\frac{\partial s}{\partial c^2}\boldsymbol{c}\cdot\partial_t\boldsymbol{c} - \boldsymbol{J}^s\cdot\boldsymbol{\nabla}T \geqslant 0. \tag{8}$$

Since the positiveness of $T\sigma^s$ forbids the presence of the divergence term in (8), we find directly that

$$\boldsymbol{J}^s = \frac{\lambda}{T}\boldsymbol{c}, \tag{9}$$

which is the well-known expression for $\boldsymbol{J}^s$, namely $\boldsymbol{J}^s = \boldsymbol{q}/T$.

Expression (8) for σ^s reduces then to

$$T\sigma^s = \boldsymbol{c}\cdot\left(2T\frac{\partial s}{\partial c^2}\partial_t\boldsymbol{c} - \frac{\lambda}{T}\nabla T\right) \geqslant 0. \tag{10}$$

The positiveness of $T\sigma^s$ is satisfied if

$$\partial_t\boldsymbol{c} + \frac{1}{z}\nabla T = -\frac{T\xi}{z\lambda}\boldsymbol{c} \qquad (\xi > 0), \tag{11}$$

wherein we have defined z through

$$\frac{\partial s}{\partial c^2} = -\frac{z\lambda}{2T^2} \tag{12}$$

and ξ is a positive coefficient as directly seen by introducing (11) in (10). In terms of $\boldsymbol{q}$, relation (11) can be written as

$$\partial_t\left(\frac{\boldsymbol{q}}{\lambda}\right) + \frac{1}{z}\nabla T = -\frac{T\xi}{z\lambda}\left(\frac{\boldsymbol{q}}{\lambda}\right), \tag{13}$$

which is of the Grioli type, i.e.

$$\partial_t\left(\frac{\boldsymbol{q}}{\lambda}\right) + \frac{1}{z}\nabla T = -\frac{1}{z}\left(\frac{\boldsymbol{q}}{\lambda}\right), \tag{14}$$

under conditions where the following identification holds:

$$\xi = \frac{\lambda}{T}. \tag{15}$$

All the coefficients appearing in (14) are generally temperature dependent. In the case of a constant value of λ, expression (14) is identical to Cattaneo's relation

$$\tau \partial_t \boldsymbol{q} + \lambda \nabla T = -\boldsymbol{q}, \tag{16}$$

which allows us to identify z with the relaxation time and to confirm λ as the heat conductivity.

Imposing that s is maximum at (local) equilibrium, it follows after integration of the Gibbs equation (6) that $\partial s/\partial c^2$ is a negative quantity. Since ξ is also positive as a consequence of the second law, we see, according to (12) and (15), that

$$z > 0, \qquad \lambda > 0. \tag{17}$$

Let us end this section with some comments about the choice of the extra variable $\boldsymbol{c} = \boldsymbol{q}/\lambda$. Although in the limiting case of small values of the relaxation time τ, the ratio $\boldsymbol{q}/\lambda$ reduces to minus the temperature gradient, it is not equivalent to select ∇T as the basic variable. There are several reasons for which $\boldsymbol{c}$ is favoured over ∇T. First, the flux $\boldsymbol{c}$ is associated with well-defined microscopic operators and therefore it allows for a better comparison with statistical mechanics or kinetic theory. Second, by taking the gradients of intensive variables, one is led to diverging terms in the expressions for the constitutive equations, a result well known in kinetic theory. Third, the choice of fluxes as variables is also supported by recent theories on non-equilibrium thermodynamics like GENERIC [13, 14], from which it is found that the fluxes provide a set of more natural variables than the gradients, and this is particularly true in rheology. Whereas for slow or steady situations, the selection of fluxes or gradients is equivalent, as they are directly related, this is no longer true for fast processes for which the use of fluxes is called for.

3. Some results on wave propagation

We now study wave and shock propagation in dielectric crystals, modelled by a rigid body whose relevant equations are (2) and (14) which, for convenience, are recalled here:

$$\partial_t u = -\boldsymbol{\nabla} \cdot (\lambda \boldsymbol{c}), \tag{18}$$

$$\partial_t \boldsymbol{c} = -\nabla\left(\int \frac{1}{z}\,\mathrm{d}T\right) - \frac{1}{z}\boldsymbol{c}. \tag{19}$$

Although the above results are valid whatever the temperature dependence of λ and z, we shall from now on assume that the relaxation time z is constant.

To proceed further, we need constitutive equations for u and s in terms of T and $\boldsymbol{c}$. The invertibility of the mapping $u \to T$ implies that $\partial u/\partial T$ is non-zero.

By expressing the Gibbs equation (6) in terms of T and $\boldsymbol{c}$ and invoking the integrability conditions, it is easy to show [7, 8, 15] that[4]

$$u = u_0(T) + a(T)c^2, \tag{20}$$

$$s = s_0(T) + \frac{z\lambda}{2T}\left(\frac{1}{T} - \frac{\lambda'}{\lambda}\right)c^2, \tag{21}$$

wherein $a(T)$ stands for

$$a = \frac{z\lambda}{2}\left(\frac{2}{T} - \frac{\lambda'}{\lambda}\right). \tag{22}$$

The equilibrium entropy $s_0(T)$ is linked to the equilibrium internal energy $u_0(T)$ by means of the Gibbs relation at equilibrium, from which it follows that

$$s_0' = \frac{1}{T}u_0' > 0. \tag{23}$$

[4] A prime denotes differentiation with respect to the only field variable upon which the function depends.

Furthermore, it is shown in [15] that the set of equations (18) and (19) is hyperbolic if the following relation is satisfied:

$$\frac{\partial u}{\partial T} = u_0' + a'c^2 > 0. \tag{24}$$

By defining $\partial u/\partial T$ as the heat capacity away from equilibrium, we may therefore state, by comparing with the equilibrium case, that a positive non-equilibrium heat capacity implies hyperbolicity.

An upper bound for c^2 can be derived when $a(T)$ is a decreasing function of temperature; we have then

$$|\boldsymbol{c}_{max}| = \sqrt{\frac{u_0'}{|a'|}}. \tag{25}$$

Following the classical procedure (e.g. [8, 16]), it is easily checked that the characteristic polynomial allowing one to determine the non-zero speeds of propagation of weak discontinuities is given by

$$P(v) := \frac{\partial u}{\partial T} v^2 - 2\left(\lambda' - \frac{\lambda}{T}\right) c_n v - \frac{\lambda}{z} = 0. \tag{26}$$

In (26) c_n stands for $\boldsymbol{c} \cdot \boldsymbol{n}$, with $\boldsymbol{n}$ the constant unit vector normal to the smooth surface propagating through the body, the functions $T(\boldsymbol{x}, t)$ and $\boldsymbol{c}(\boldsymbol{x}, t)$ remain continuous across the moving surface but discontinuities in their first derivatives are permitted.

At equilibrium, for which $\boldsymbol{c} = \boldsymbol{0}$, the velocity of propagation is simply given by

$$v_0^2 = \frac{\lambda}{z u_0'}. \tag{27}$$

By requiring that the set is completely exceptional [18], explicit expressions for u_0 and λ were obtained in [16]. For these particular constitutive laws, the system is not only completely exceptional but also strictly exceptional [17], which means that the only possible shocks are those moving with the characteristic velocities.

4. Shock wave propagation in highly pure dielectric crystals

In this section, we study the propagation of plane shock waves travelling in a medium which is in an equilibrium state given by $T = T_0$, $\boldsymbol{c} = \boldsymbol{0}$. Consider a smooth surface propagating through the body across which the functions $T(\boldsymbol{x}, t)$ and $\boldsymbol{c}(\boldsymbol{x}, t)$ may undergo jumps.

The Rankine–Hugoniot compatibility conditions, which must be satisfied by the set (18) and (19) across the shock front, are

$$-\Sigma[u_0 + ac^2] + [\lambda c_N] = 0, \tag{28}$$

$$-\Sigma[\boldsymbol{c}] + \frac{1}{z}[T]\boldsymbol{N} = \boldsymbol{0}, \tag{29}$$

after use is made of (20), where Σ is the shock wave speed, $c_N = \boldsymbol{c} \cdot \boldsymbol{N}$, $[\alpha] = \alpha - \alpha_0$ denotes the jump between the limiting values of a generic quantity α in the perturbed state (α) and in the unperturbed state (α_0) across the shock wavefront, $\boldsymbol{N}$ is the unit vector normal to the shock front. Expressions (28) and (29) can be considered as an algebraic set of four scalar equations for the five unknowns T, $\boldsymbol{c}$ and Σ in terms of the assigned unperturbed field (T_0, $\boldsymbol{0}$). By taking the perturbed temperature T as a shock parameter, we obtain from (28) and (29)

$$\Sigma^2(T_0, T) = \frac{z\lambda - a(T - T_0)}{z^2\{u_0(T) - u_0(T_0)\}}(T - T_0), \tag{30}$$

$$\boldsymbol{c}(T_0, T) = \frac{1}{z\Sigma}(T - T_0)\boldsymbol{N}. \tag{31}$$

It is well known that, among the mathematical solutions of the Rankine–Hugoniot equations, only the stable ones are physically admissible. In the following, we consider as criteria for selecting physical shocks both the Lax condition [18] and the condition of entropy growth across the shock [19].

Lax conditions. The Lax criterion for selecting physical shocks states that the admissible shocks are those for which the shock velocity is greater than the unperturbed characteristic velocity and less than the perturbed one. In our case, choosing $\Sigma > 0$, the Lax conditions read as

$$0 < v_0(T_0) < \Sigma(T_0, T) < v(T_0, T), \qquad \lim_{\Sigma \to v_0(T_0)} T = T_0. \tag{32}$$

The expressions for the unperturbed characteristic velocity $v_0(T_0)$ and of the perturbed one $v(T_0, T)$ are, taking (26), (27) and (31) into account,

$$v_0(T_0) = \sqrt{\frac{\lambda(T_0)}{z u_0'(T_0)}}, \tag{33}$$

$$v(T_0, T) = \frac{(\lambda' - \lambda/T)\beta + \sqrt{\{(\lambda' - \lambda/T)\beta\}^2 + (\lambda/z)(u_0' + a'\beta^2)}}{u_0' + a'\beta^2} \tag{34}$$

wherein β stands for

$$\beta = \frac{T - T_0}{z\Sigma}. \tag{35}$$

To illustrate numerically the temperature windows wherein the Lax rule is satisfied, we will use some experimental data on heat pulse propagation in NaF and Bi crystals. This requires us to specify the forms of the constitutive equations for energy at equilibrium and for the heat conductivity.

For crystalline dielectrics at low temperature, one satisfactory relation which links the equilibrium energy to the absolute temperature T is Debye's law:

$$u_0 = \tfrac{1}{4}\epsilon T^4, \tag{36}$$

where $\epsilon = 2.3$ J m^{-3} K^{-4} for NaF [20] and $\epsilon = 55$ J m^{-3} K^{-4} for Bi [21].

It is important to observe that the expression (36) does not make the system strictly exceptional [16].

Moreover, it is a simple matter to obtain the expression for λ/z in terms of T, once the characteristic speed at equilibrium v_0 is known. Indeed, from (27), it is directly seen that

$$\frac{\lambda}{z} = u_0' {v_0}^2. \tag{37}$$

The temperature dependence of the speed of propagation v_0 measured by Jackson *et al* [5] (in NaF) and Narayanamurti and Dynes [6] (in Bi), is well described by the empirical relation [22]

$$v_0^2 = \frac{1}{A + BT^n}, \tag{38}$$

where A, B and n are constants. By taking (38) into account in (37), we obtain

$$\frac{\lambda}{z} = \frac{u_0'}{A + BT^n}. \tag{39}$$

It is important to stress that in the Cattaneo model wherein λ/z is a constant, assuming (36), expression (39) cannot be satisfied. As a consequence, it may be said that the usual Cattaneo equation is not appropriate for describing waves and shocks at very low temperature.

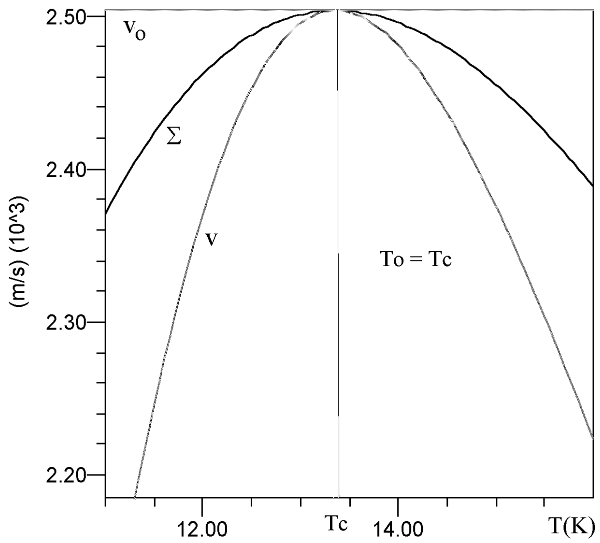

Figure 1. Σ and v versus T for NaF ($T_0 = T_c$).

The values of A, B and n in (38) giving a good fit [23] with the measured values of v_0 are for NaF

$$A = 9.09 \times 10^{-8}\ \mathrm{SI}, \qquad B = 2.22 \times 10^{-11}\ \mathrm{SI}, \qquad n = 3.1 \tag{40}$$

and for Bi

$$A = 9.07 \times 10^{-7}\ \mathrm{SI}, \qquad B = 7.58 \times 10^{-9}\ \mathrm{SI}, \qquad n = 3.75. \tag{41}$$

Of course, these data relate to the temperature range in which the second sound is observed, that is 10 K $\leqslant T \leqslant$ 18.5 K for NaF and 1.4 K $\leqslant T \leqslant$ 4 K for Bi.

After substituting (36) and (39) in (30) and (34), we are able to determine the values of Σ and v as a function of the perturbed temperature T (behind the shock), for a fixed value of the equilibrium temperature T_0 (ahead of the shock) for both NaF and Bi. For brevity, the figures will be only for NaF, as the behaviour of Bi is very similar.

As indicated in figure 1, the main result is the existence of a critical temperature T_c, such that when $T_0 = T_c$, no shock is possible. Such a situation is similar to that found by Ruggeri *et al* [24].

These authors have also demonstrated [25] that the critical temperature T_c corresponds to a minimum of the so-called *shape function*. By following the same procedure as in [25], it is found that the *shape function* Ψ is here given by

$$\Psi = \frac{1}{T} v_0^3 u_0', \tag{42}$$

which, according to (36) and (38), has a unique minimum at

$$T_c = \left\{ \frac{4A}{(3n-4)B} \right\}^{1/n}. \tag{43}$$

It is found that $T_c = 13.36$ K for NaF and $T_c = 3.06$ K for Bi.

As observed by Ruggeri *et al* [25], T_c defines the boundary between two very different phenomena: *hot shocks* and *cold shocks*. Consider first the case $T_0 < T_c$ and $T_0 < T < T_L$ with T_L the maximum temperature value obeying Lax criterion (T_L is a function of T_0): then

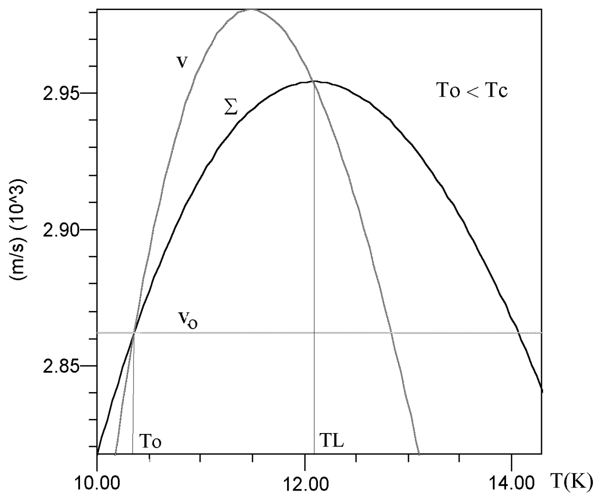

Figure 2. Σ and v versus T for NaF ($T_0 < T_c$).

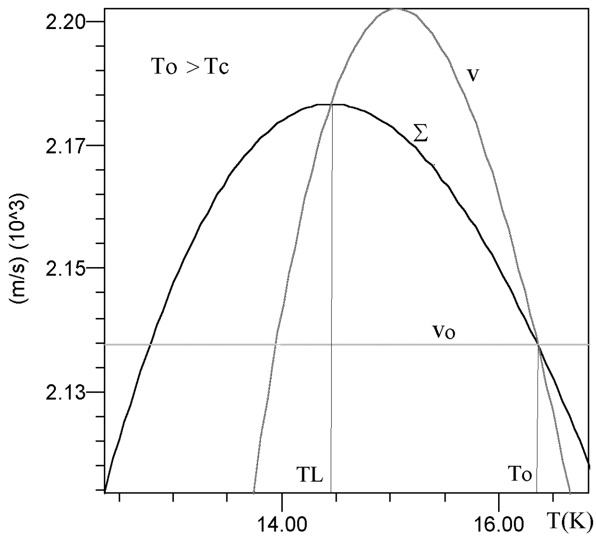

Figure 3. Σ and v versus T for NaF ($T_0 > T_c$).

only *hot shocks* are possible (see figure 2); hot shocks are thus characterized by a heating of the crystal. In contrast, for $T_0 > T_c$ and $T_L < T < T_0$, *cold shocks* are predicted (see figure 3); in this case the shock wave produces a cooling of the body and the temperature jumps down after the passage of the wave.

Entropy growth across the shock. For the model under consideration, the expression for the entropy production σ^s across the shock is

$$\sigma^s(T_0, T) = \Sigma[s] - [\boldsymbol{J}^s] \cdot \boldsymbol{N} \tag{44}$$

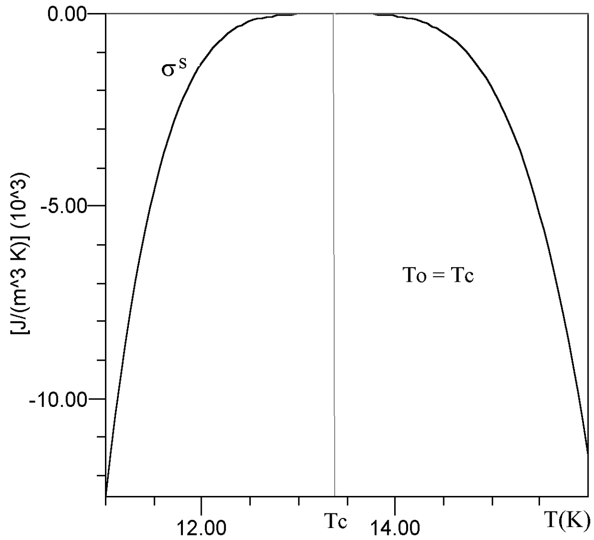

Figure 4. σ^s versus T for NaF ($T_0 = T_c$).

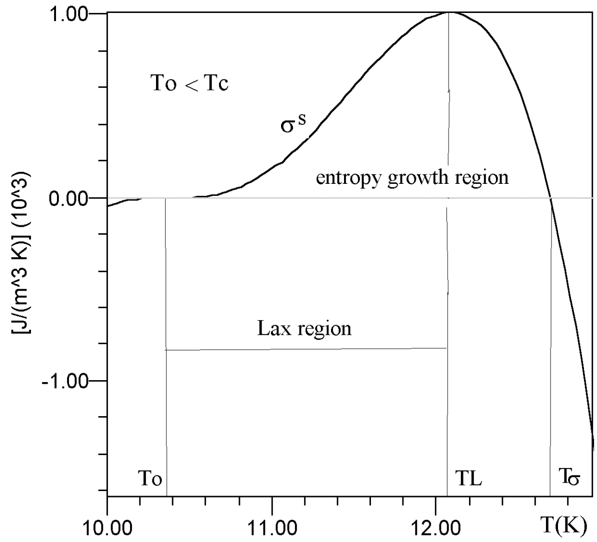

Figure 5. σ^s versus T for NaF ($T_0 < T_c$).

and, by virtue of the previous results,

$$\sigma^s(T_0, T) = \Sigma\left[s_0(T) + \frac{z\lambda}{2T}\left(\frac{1}{T} - \frac{\lambda'}{\lambda}\right)c^2\right] - \left[\frac{\lambda}{T}c_N\right]. \tag{45}$$

Thermodynamics requires that production across the shock is positive definite, i.e.

$$\sigma^s(T_0, T) > 0 \tag{46}$$

and this criterion is usually referred to as the entropy growth condition. The numerical evaluations of σ^s are reported in figures 4–6, for the same choice of equilibrium temperature T_0 as for the Lax criterion.

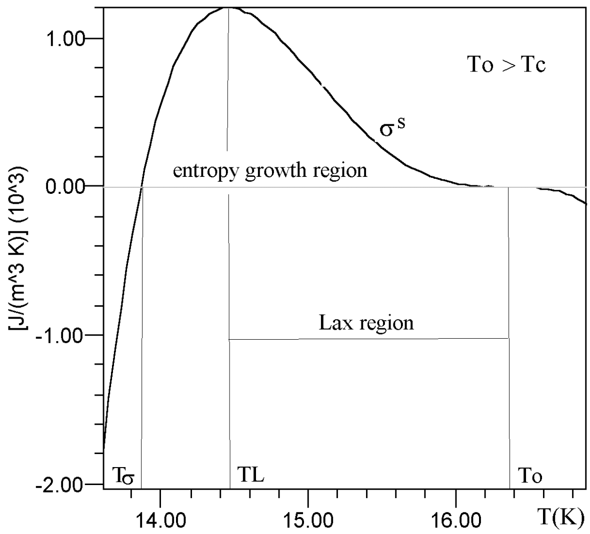

Figure 6. σ^s versus T for NaF ($T_0 > T_c$).

Figure 4 confirms that no shock is admissible when $T_0 = T_c$. A comparative analysis—between figures 2, 3 and figures 5, 6—shows that the range of temperature where the entropy growth condition is satisfied is larger than the domain where the Lax rule holds. However, if we accept that the entropy growth condition is only applicable for shocks passing through the null shock [25], then the two criteria coincide (σ^s has a maximum at T_L).

A last remark is in order. In Ruggeri *et al* [25] it was shown that the temperature range in which the entropy growth condition is valid coincides with the region whose limit temperatures are those for which the perturbed characteristic velocity is equal to the unperturbed one. Quoting [25]: '*It does not seem easy to verify that this property remains valid also in the general theory of shock waves*'. Our analysis offers an example in which this property is not satisfied. Indeed, it is observed from figure 7, where Σ, v and σ^s are plotted versus T, that the temperatures T_σ and T_n (which depends on T_0) do not coincide: T_n is the temperature at which both v and v_0 are equal, while T_σ is the extremum temperature defining the domain of validity of the entropy growth criterion. Conclusions similar to these of figure 7 for which $T_0 < T_c$ have been obtained for $T_0 > T_c$.

5. Conclusions

In this paper we have studied the shock wave propagation in dielectric crystals at low temperature (<20 K). The mathematical model is inspired by EIT wherein the dissipative fluxes are introduced as extra variables. Here, instead, we use a weighted flux, wherein the 'weighted' coefficient is related to the material properties of the process, like heat conductivity.

The present model is equivalent to Grioli's one and is an extension of the more classical descriptions offered by Cattaneo's equation and Fourier's law.

Following the classical procedure for shock wave propagation in hyperbolic systems, and using experimental data obtained from heat pulse experiments on NaF and Bi at low temperature, we arrive at the conclusion that formation of shock waves in NaF and Bi is possible at low temperature. But the remarkable fact is that not only usual shocks (hot shocks) but also a new family of shocks (so-called cold shocks) are predicted. The latter are rather

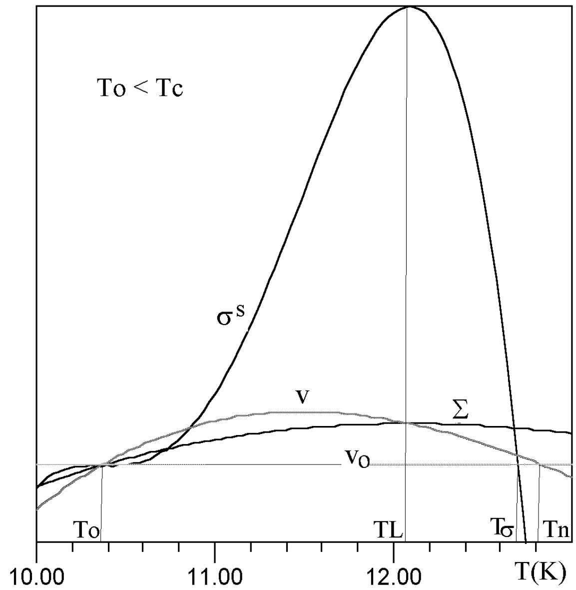

Figure 7. Σ, v and σ^s versus T ($T_0 < T_c$).

particular, as their effect is to lower the temperature after the passage of the front. This is a consequence of the hypotheses underlying our model, namely the dependence of internal energy and entropy on the extra variable $\boldsymbol{q}/\lambda$, besides the temperature. As a result, the entropy will grow even if the temperature behind the front wave is smaller than that ahead, as explicitly exhibited by figures 3 and 6. The presence of cold shocks is not usual, but was previously noticed by Müller and Ruggeri [26] in monatomic gases and by Ruggeri *et al* [25] in the analysis of shock waves in NaF and Bi. At this point it is worth stressing the differences between the present model and the Ruggeri *et al* [25] analysis. The first difference lies in the choice of the extra variable: instead of the heat flux $\boldsymbol{q}$, we have selected $\boldsymbol{c} = \boldsymbol{q}/\lambda$. But the main difference is that in [25], the internal energy is assumed to be only temperature dependent. Another difference concerns the values of the critical temperature separating the regimes of hot and cold shocks. We find that for NaF the critical temperature is $T_c = 13.36$ K ($T_c = 15.36$ K in [25]) while for Bi, $T_c = 3.06$ K ($T_c = 3.38$ K in [25]). Although these values are rather close, there is nevertheless a difference (more or less) of about 10%. It is also interesting to observe that our values are closer to the experimental values of the temperature at which second-sound pulses are appearing [27]. Finally, and in contrast with previous findings, some transport coefficients such as heat conductivity and heat capacity are no longer constant but are now temperature dependent, showing that the results presented here cover a wide class of phenomena. In particular, the present model is applicable to all relaxation processes and not only to low-temperature situations.

Acknowledgments

One of us (GL) wishes to thank the members of the Dipartimento di Matematica e Informatica dell'Università di Catania for their warm hospitality. This work was supported by CNR through GNFM and by MURST, Project: *Non-Linear Mathematical Problems of Wave Propagation and Stability in Models of Continuous Media*. Support from SSTC-Belgium, PAI programme, under contract No IV 06 is also acknowledged.

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